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Dependence of turbulent mixing on the initial roughness in the evaporation front with continuous density profile at the interface

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An analytical solution describing turbulent mixing at the ablation front of fusion targets as well as mixing under the initial continuous density profile of the interface within the limits of the semiempirical model has been obtained. The mixing is the result of the instability arising when the pressure gradient direction relative to the density gradient is such that $(\partial p/\partial x)(\partial \rho/\partial x)<0$. In the first case, the ablation front decreases the turbulent mixing role. The critical value of the initial roughness $h_{\rm cr}$ under which the full mixing of the shell takes place is found. In the classical case the shell will be fully mixed under $A_{\rm inf}\geq 14$ independent of the initial roughness h. The ablation will remove this restriction. For any aspect ratio $A_{\rm inf}$ there is the roughness $h_{\rm cr}$ and if $h_0<h_{\rm cr}$ then the shell will be mixed only partially. In the second case, the formula connecting the time of delay in the interface mixing with the width of the initial continuous density profile and the initial perturbations size is derived. A formula is obtained for the arbitrary Atwood number α_A. The processing of the earlier conducted experiments of Yu. A. Kucherenko [R. J. Ardashova *et al.*, Vopr. At. Nauki Tekh. Ser. Teor. Prikl. Fiz. **1**, 20 (1989)] has indirectly verified the validity of the K-model constant. In both cases the essential dependence of the arising turbulent mixing on the initial roughness sizes ($h_0=L_0$) has been revealed. If in the classical case there is the nontrivial solution for zero roughness under the Rayleigh-Taylor mixing, then in the cases considered here just the trivial solution meets zero roughness. A similar dependence on the initial roughness has been observed previously [V. E. Neuvazhaev, Russ. J. Math. Sim. **3**, 10 (1991)] under mixing induced by the action of impulsive acceleration (the Richtmyer-Meshkov mixing).

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I. INTRODUCTION

By using a simple diffusion model similar to the model proposed by Belen'ky and Fradkin [2] in the work of Takabe and Yamamoto [1] it was shown that allowance for the stabilizing effect of the ablation front leads to a significant decrease of turbulent mixing. This is connected with the fact that the perturbation growth increment, obtained previously [3], is such that there is a critical wavelength and the amplitude of perturbations at the front of ablation is not growing when the wavelength is less than the critical one.

However, the authors [1] have not, evidently, noticed that in this case, as opposed to the classical one, the essential dependence of the mixing width on the initial roughness (an initial perturbation level) occurs. The analogous problem arises in the following cases: (1) when the initial density profile is continuous, and acceleration is constant; (2) when the initial density profile is discontinuous, and acceleration is impulsive [5] (Richtmyer-Meshkov instability). In this case, for the initial roughness equal to zero the trivial solution is obtained. Therefore, the final result should be formulated as the requirement for the initial roughness of the target under which the target shell is not disrupted.

In the present work the analytical dependence of the permissible initial perturbations on the aspect ratio $A_{\rm inf}$ has been obtained within the limits of the model [2]. Section II is devoted to the derivation of this formula.

In Sec. III the connection is established between the discussed problem and the results of Ref. [4] concerning the experimental determination of the stabilizing effect produced by the initial continuous density profile of the interface. Within the limits of the K model [6] an analytical formula which determines the relation of the delay in mixing versus the initial roughness is built,

$$\left[\frac{L_c}{L_0}\right]^{1/2}=\exp\left[2\eta_1\alpha_1\left[\frac{\Phi(\eta_1)2S_c\alpha_A}{(1+2k_0)L_c}\right]^{1/2}\right], \qquad (1)$$

where L_0 $(=2h_0)$ and L_c are the initial roughness and width of the initial continuous density profile of the boundary, respectively, and S_c is the displacement of the system at the instant of time when the turbulent mixing width L is equal to L_c. Here, the formula is given for the K model. $\eta_1,\alpha_1,\Phi(\eta_1)$, k_0 are constants determined below. The formula for the KE model [7] has a form analogous to the constants of the KE model.

Formula (1) was used for processing the experiments [4]. The results of processing corroborated the theoretical curve slope calculated on the basis of previously determined α_1, ν, and $C_\mu, C_{\varepsilon 1}, C_{\varepsilon 2}$ in the K and KE models, respectively.

II. TURBULENT MIXING AT THE ABLATION FRONT

In Ref. [1] the following approximate statement of the problem is considered. Let there be a target of radius R_0 with shell thickness ΔR. The external side of the shell is exposed to radiation, which leads to compression and displacement of the shell to a distance equal to half its radius R_0. These factors determine the instant of time t_A in which the shell state is observed:

$$gt_A^2=R_0 , \tag{2}$$

where g is the acceleration acting on the shell, and the action of the shock waves should be taken into account by changing the thickness ΔR into a new thickness $\Delta R_{\rm inf}$. The latter is assumed to be constant in the considered time interval $(0,t_A)$. The Atwood number α_A characterizing the density relation at the ablation front is believed to be equal to 1: $\alpha_A=1$.

The turbulent mixing influence on the external side of the shell as a result of the Rayleigh-Taylor instability is studied. If one applies the known law of mixing [8,9],

$$h(t)=0.07\alpha_A g t^2 , \tag{3}$$

then the condition for the full mixing of the shell up to the instant $t=t_A$ will be the following:

$$h(t_A)=\Delta R_{\rm inf} . \tag{4}$$

Equations (3) and (4) lead to limitation by an aspect ratio $A_{\rm inf}$:

$$A_{\rm inf}=\frac{R_0}{\Delta R_{\rm inf}} , \quad A_{\rm inf}<\frac{1}{0.07} .$$

When performing the last inequality, an incomplete mixing of the shell is possible; otherwise, it gets mixed up completely and repeatedly.

However, the displacement of the ablation front with mass velocity V_0 leads to the stabilization of short-wave perturbations. Increment γ is presented in the form [3]

$$\gamma=\alpha\sqrt{kg}-\beta k V_0 , \tag{5}$$

where k is the wave number and $\alpha=0.9$, $3\le\beta\le4$. For determining the velocity V_0 one uses the relation

$$V_0 t_A=f\Delta R_{\rm inf} ,$$

which means that at the instant t_A the fth part of the shell has evaporated. It is further convenient to go to nondimensional variables:

$$\tilde h=\frac{h}{R_0} , \quad \tilde x=\frac{x}{R_0} , \quad \tilde t=\frac{t}{t_A} .$$

In [2], for the estimation of the mixing influence at the front of ablation, the diffusion model similar to [1] is suggested for use. Then the equation for turbulized density ρ_0 is taken in the form

$$\frac{\partial\rho_0}{\partial\tilde t}=\frac{\partial}{\partial\tilde x}\tilde D\frac{\partial\rho_0}{\partial\tilde x} , \tag{6}$$

where $\rho_0=\rho_0(x,t)$ is the average density value in the direction of the axis, y being orthogonal to the axis x: $\rho_0=\langle\rho\rangle_y$ as well as

$$\tilde D=\alpha b^{1/2}a^2\times\begin{cases}\tilde h^{3/2}-b^{1/2}\dfrac{\tilde h}{\alpha s} , & \text{for } \tilde h>\dfrac{b}{l_m} ,\\[2mm] \dfrac{b^{-1/2}}{4}\alpha s\tilde h^2 , & \text{for } \tilde h<\dfrac{b}{l_m} ,\end{cases} \tag{7}$$

$$l_m=\left[\frac{\alpha A_{\rm inf}}{2\beta f}\right]^2 , \quad s=\frac{A_{\rm inf}}{\beta f} .$$

Here a and b are constants that are determined below. It should be noted that in [2], formula (7) is given with misprints.

The diffusion coefficient $\tilde D$ is only the function of the time; therefore, as in [10], it is convenient to go to a new variable:

$$\tilde D\partial\tilde t=\partial\tau . \tag{8}$$

Then the solution of Eq. (6) can be presented through the integral of probability Φ:

$$\rho_0=\frac{\rho_a}{2}\left[1+\Phi\left[\frac{\tilde x}{2\sqrt{\tau}}\right]\right] , \quad \Phi(\eta)=\frac{2}{\sqrt{\pi}}\int_0^\eta e^{-z^2}dz . \tag{9}$$

Here, as well as in [1], the maximum density at the ablation front is symbolized by ρ_a.

The solution (9) is true for an infinite medium; however, it can be used for the shell as well. The front of the mixing is effectively determined by substituting the density profile in the mixing region by the linear function

$$\rho=\frac{\rho_a\tilde x}{2h} ,$$

but only so that the mixed matter mass determined according to the density profile (9) has been conserved. This leads to the following expression for $\tilde h$ [5]:

$$\tilde h=2\eta_1\sqrt{\tau} , \quad \eta_1=\frac{2}{\sqrt{\pi}} . \tag{10}$$

From Eqs. (7), (8), and (10), the equation for width $\tilde h$ is as follows:

$$\frac{d\tilde h}{d\tilde t}=2a^2\eta_1^2\begin{cases}\dfrac{\alpha^2}{4}s\tilde h , & \tilde h<\dfrac{4b}{\alpha^2 s^2} ,\\[2mm] \alpha\sqrt{b}\sqrt{\tilde h}-\dfrac{b}{s} , & \tilde h>\dfrac{4b}{\alpha^2 s^2} .\end{cases} \tag{11}$$

Integration of (11) leads to the following solution:

$$\sqrt{\tilde h}=\begin{cases}\sqrt{\tilde h_0}\exp\left[\left[\dfrac{\alpha_0}{b}\right]^{1/2}\dfrac{\alpha s}{4}\tilde t\right] & \text{if } \tilde h<\dfrac{4b}{\alpha^2 s^2} ,\\[2mm] \sqrt{\alpha_0}(\tilde t-\tilde t_c)+\dfrac{\sqrt{b}}{\alpha s}\left\{2-\ln\left[\left[\dfrac{\tilde h}{b}\right]^{1/2}\alpha s-1\right]\right\} , & \text{if } \tilde h>\dfrac{4b}{\alpha^2 s^2} ,\end{cases} \tag{12}$$

where

$$\sqrt{\alpha_0}=a^2\eta_1^2\alpha\sqrt{b}=\sqrt{0.07}\ ,$$

$$\tilde{t}_c=\frac{\sqrt{b}}{\alpha\sqrt{\alpha_0 s}}\ln\frac{16b^2}{\alpha^4 s^4\tilde{h}_0^2}\ .$$

In the classical case when ablation is absent ($V_0=0$) the solution has the form

$$\sqrt{\tilde{h}}=\sqrt{\tilde{h}_0}+\sqrt{\alpha_0}\tilde{t}\ . \tag{13}$$

We pay attention to the dependence of the solution of Eq. (12) on the initial roughness $\tilde{h}_0$, but, if in the classical case (13) the nontrivial solution exists when roughness is zero, then in case (12) only the zero solution corresponds to a zero roughness. Therefore, in the case of the stabilizing effect of the ablation front the dependence appears on one more parameter—the value of the initial roughness $\tilde{h}_0$. The final result may be formulated as the requirement of the initial roughness, which does not result in the full mixing of the shell. The boundary value for roughness as the function of $A_{\rm inf}$ (or s) is obtained from (12), assuming that $\tilde{t}=1$,

$$\tilde{h}=\frac{\Delta R}{R_0}=\frac{1}{A_{\rm inf}}=\frac{1}{s\beta f}\ .$$

Finally, we have

$$\tilde{h}_{\rm cr}=\begin{cases} 1\Big/\left\{A_{\rm inf}\exp\left[\left[\dfrac{\alpha_0}{A^*_{\rm inf}}\right]^{1/2}A_{\rm inf}\right]\right\} & \text{for } A_{\rm inf}<A^*_{\rm inf}\ ,\\[2ex] \dfrac{A^*_{\rm inf}\exp\left[\left[\dfrac{A_{\rm inf}}{A^*_{\rm inf}}\right]^{1/2}[1-\sqrt{\alpha_0 A_{\rm inf}}]-1\right]}{A^2_{\rm inf}\left\{\left[2\left[\dfrac{A_{\rm inf}}{A^*_{\rm inf}}\right]^{1/2}-1\right]\right\}^{-1/2}} & \text{for } A_{\rm inf}>A^*_{\rm inf}\ , \end{cases} \tag{14}$$

$$A^*_{\rm inf}=\frac{4\beta^2 f^2 b}{\alpha^2}\ .$$

If we take constants suggested in [2], $\alpha_0=0.07$, $\alpha=0.9$, $\beta=3.5$, $f=\frac{2}{3}$, $b=3$, then we obtain $A^*_{\rm inf}=80$. The dependence of the initial roughness on the aspect ratio $A_{\rm inf}$ is depicted in Fig. 1. When $A_{\rm inf}<A^*_{\rm inf}$, the curve has a smoother character than for $A_{\rm inf}<A^*_{\rm inf}$. For the classical increment (a dot-dash curve) the full mixing takes place just at $A_{\rm inf}=14$. In the case of increment (5) a small initial roughness at which the shell is not totally displaced is possible according to formula (14) for an arbitrary large value of the aspect ratio. Thus, for rather large values of $A_{\rm inf}$ the following asymptotic formula is true:

$$\tilde{h}_{\rm cr}=\frac{(A^*_{\rm inf})^{3/4}}{\left\{\sqrt{2}A^{7/4}_{\rm inf}\exp\left[A_{\rm inf}\left[\dfrac{\alpha_0}{A^*_{\rm inf}}\right]^{1/2}\right]\right\}}\ .$$

It is quite another matter that such requirements cannot be really fulfilled. In the same Fig. 1, dependencies for $\beta=3$ (a lower curve) and $\beta=4$ (an upper curve) are presented by dashed curves.

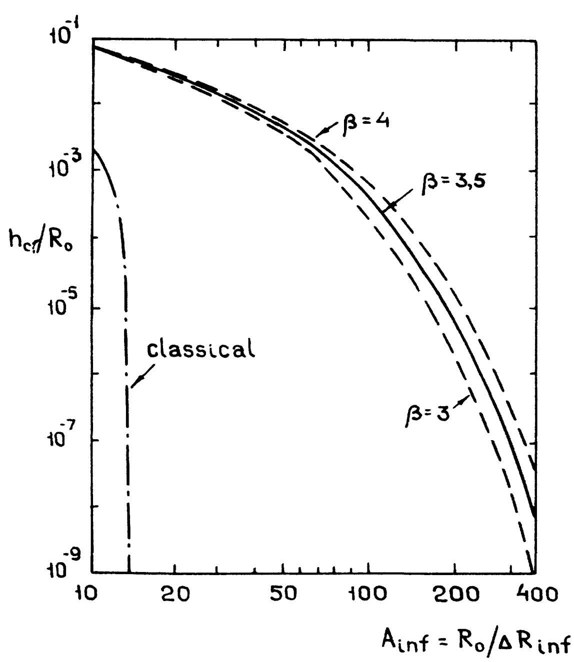

FIG. 1. Dependence of the relative critical initial roughness $h_{\rm cr}/R_0$, which results in full mixing of the shell, on the aspect ratio $A_{\rm inf}$.

III. DELAY IN TURBULENT MIXING WITH CONTINUOUS DENSITY PROFILE OF THE INTERFACE

The problem described in this section is closely connected with the problem considered above. Let there be two incompressible fluids of densities ρ_1 (a light one) and ρ_2. Constant acceleration g is directed so that the Rayleigh-Taylor instability takes place. It is known that under the initial continuous density profile of the interface, the delay in the turbulent mixing evolution takes place. This problem has been studied experimentally in Ref. [4] where for $\rho_2/\rho_1=4$ the dependence of the delay on the value of random initial perturbations specified in the middle of the layer with continuous density profile has been established.

Now we consider the same problem within the limits of the K model by applying the approximate approach set forth in [5,10]. For mixture density ρ and kinetic energy $\overline{v}^2/2$ we have

$$\frac{\partial\rho}{\partial t}=\frac{\partial}{\partial X}D\frac{\partial\rho}{\partial X}\ , \tag{15}$$

$$\frac{\partial\rho\overline{v}^2}{2\partial t}+\frac{\nu\rho D\overline{v}^2}{\alpha_1^2L^2}=\rho D\overline{\gamma}^2\ , \tag{16}$$

$$D=\alpha_1L\overline{v}\ ,$$

$$\overline{\gamma}^2=\begin{cases}\left.\dfrac{\overline{\partial\rho}}{\rho\partial X}\right|_{t=0}g & \text{if } L<L_c\ ,\\ \dfrac{\overline{\partial\rho}}{\rho\partial X}g & \text{if } L>L_c\ .\end{cases}$$

An alternative way used for obtaining Eq. (16), as compared with averaging performed previously in [5], consists in the averaging (a line from above) of the right side. The initial profile of density can be arbitrary. However, it can be approximately substituted by the profile

$$\rho(0,X)=\frac{\rho_1+\rho_2}{2}+\frac{\rho_2-\rho_1}{2}+\Phi\left[\frac{2X}{L_c}\right]\ .$$

Then Eqs. (15) and (16) are integrated:

$$\overline{v}^2=\begin{cases}\dfrac{\Phi(\eta_1)g\alpha_AL^2}{4\eta_1^2L_c(1+2k_0)} & \text{if } L<L_c\\ \overline{v}_c^2+\dfrac{\Phi(\eta_1)g\alpha_AL}{2\eta_1(1+4k_0)}\left[1-\left[\dfrac{L_c}{L}\right]^{1+4k_0}\right] & \\ \text{if } L>L_c\ . \end{cases} \tag{17}$$

Here,

$$\overline{v}_c^2=\frac{\Phi(\eta_1)g\alpha_A}{4\eta_1^2(1+2k_0)}L_c\ ,$$

$$k_0=0.25+\frac{\nu}{16\eta_1^2\alpha_1^2}+\frac{\Phi(\sqrt{2}\eta_1)}{12\sqrt{2}}\alpha_A^2\ .$$

The equation for width L is derived on the basis of the following relations:

$$\delta\tau=D\delta t\ ,\quad L=4\eta_1\sqrt{\tau}\ ,\quad \frac{dL}{dt}=8\eta_1^2\alpha_1\overline{v}\ . \tag{18}$$

The combined consideration of (17) and (18) leads to the following solution:

$$\sqrt{L}=\sqrt{L_0}\exp\left[2\eta_1\alpha_1\left[\frac{\Phi(\eta_1)g\alpha_A}{(1+2k_0)L_c}\right]^{1/2}t\right]\ ,\quad L<L_c$$
$$\sqrt{L}\simeq\sqrt{L_c}+\alpha_1\eta_1\left[\frac{8\Phi(\eta_1)g\alpha_A}{(1+4k_0)}\right]^{1/2}(t-t_c)\ ,\quad L>L_c\ . \tag{19}$$

Here, for $L>L_c$ the approximate presentation of the solution is given insofar as the accurate solution has a cumbersome expression, and for further purposes it will not be required.

We pay attention to the dependence of the solution on the initial roughness L_0. As in the previous section, the zero solution meets the zero roughness.

We obtain the formula for the value of delay in turbulent mixing due to the initial roughness. It is derived from (19) if the delay instant is determined by the equality

$$L=L_c\ .$$

Then we have

$$\left[\frac{L_c}{L_0}\right]^{1/2}=\exp\left[2\eta_1\alpha_1\left[\frac{\Phi(\eta_1)g\alpha_A}{(1+2k_0)L_c}\right]^{1/2}t_c\right]\ . \tag{20}$$

By generalizing formula (20) for the case of slowly changing acceleration we obtain the final result in the form of (1).

As has already been noted, the dependence of the delay in mixing on the initial roughness was studied experimentally [4]. Formula (1) allows us to process experiments by passing on to the plane of the variables (x,y):

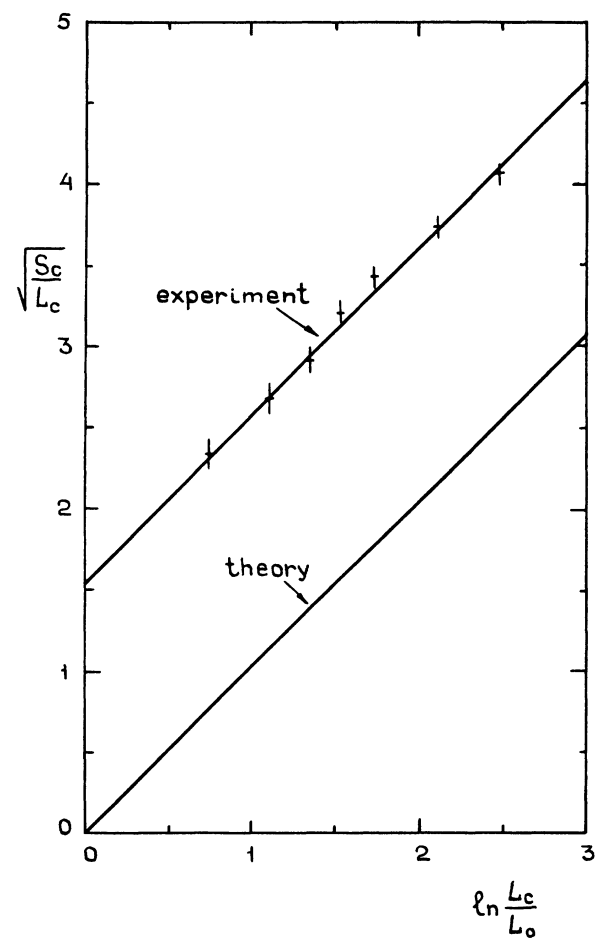

FIG. 2. Dependence of the value $(S_c/L_c)^{1/2}$ on $\ln(L_c/L_0)$, where L_c is the width of a continuous density profile, L_0 is the initial roughness, and S_c is the displacement of the ampoule, if $L=L_c$.

$$x=\ln\frac{L_c}{L_0}\ , \quad y=\left[\frac{S_c}{L_c}\right]^{1/2} .$$

In Fig. 2 the results of [4] are plotted and the inclination is determined by taking into account the dependence on the Atwood number α_A:

$$y=\frac{1.02}{\sqrt{\alpha_A}}x+1.55 . \tag{21}$$

An experimental curve, as opposed to the theoretical one, does not go through zero. We believe that it is connected with the ambiguous nature of the initial roughness determination in the experiment.

Now we shall determine the inclination according to formula (1) and constants α_1,ν chosen previously in [5,10], in order to describe the generally adopted law (3) under constant acceleration and "the law $\frac{2}{7}$" under removed acceleration. For this we shall assume

$$k_0(\alpha_A=0)=0.25+\frac{\nu}{16\eta_1^2\alpha_1^2}=1.25 , \tag{22}$$

$$\frac{4\eta_1\alpha_1\sqrt{\Phi(\eta_1)}}{\sqrt{1+4k_0(0)}}=\sqrt{0.28} .$$

Substitution of (22) into (1) leads to the formula

$$y=\frac{1.02}{\sqrt{\alpha_a}}x .$$

The same result has been obtained under processing of experiments [4] [see (21)]. Thus the indirect substantiation of the correctness of the choice of α_1 and ν constants made above has been obtained.

If after rather a long time the mixing region width obtained with a continuous density profile is compared with that without an initial continuous density profile but with the same initial roughness, then it is evident that, in the case of the initial continuous density profile, the mixing region width will always be less. However, if in the case having a discontinuous density profile the initial roughness is infinitesimal, then this will not be true. The mixing region width with the initial continuous density profile may appear to be larger if the relative initial roughness L_0/L_c is sufficiently great.

We find the critical value at which the width will be the same in these two cases. For this purpose we make use of the solution for the case of discontinuous initial density and zero initial roughness:

$$\sqrt{L}=\alpha\eta_1\left[\frac{8\Phi(\eta_1)\alpha_A g}{(1+4k_0)}\right]^{1/2} t .$$

Setting $L=L_c$ we substitute t_c from Eq. (20) into this formula:

$$\left[\frac{L_c}{L_0}\right]_{\rm cr}=\exp\left[\frac{2(1+4k_0)}{1+2k_0}\right]^{1/2}=6.37 .$$

IV. CONCLUSION

The process of turbulent mixing induced by the Rayleigh-Taylor instability is mainly determined by the increment γ. If the increment is limited for short waves, then this leads to slowing down in the turbulent mixing evolution. Such slowing down arises at the front of ablation when irradiating the shell targets and in the case of the continuous density profile of the interface. The semiempirical theories of the diffusion type allows us to make a quantitative estimation in the form of an analytical formula connecting the mixing region width with the initial roughness value. The increment which determines the turbulent diffusion coefficient D is found. When the increment is limited for short-wave perturbations the diffusion coefficient dependence on the length scale ($L=h$) increases from $D\sim h^{2/3}$ (the classical case) to $D\sim h^2$ (the considered cases). This, in its turn, changes the dependence on the initial data. The solution is such that the infinitesimal width of mixing corresponds to the infinitesimal initial roughness.

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